

First Exam MTH 221 , Fall 2011

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QUESTION 1. Each = 2.5 points, Circle the correct answer

(i) One of the following matrices is non-invertible (singular)

a) $\begin{bmatrix} 3 & -9 \\ 2 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 8 & -12 \\ -4 & 6 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 0 & 0 \\ -2 & -1 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ d) None of the previous is correct.

(ii) If A is a 3×3 matrix and $det(A) = 2$ and $det(A^2 - A) = 4$, then $det(A - I_3) =$

a) 1 b) 2 c) 3 d) Cannot be determined

(iii) If A is a 3×3 matrix such that $det(A) = 6.3$, then one of the following statements is correct about the system $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

a)It is possible that $x_1 = x_2 = x_3 = 1$ is a solution to the system.

b) It is possible that the system has infinitely many solutions.

c) $x_1 = x_2 = x_3 = 0$ is the only solution to the system

d) (a) and (b) are correct.

(iv) Given $A \quad \overrightarrow{2R_1} \quad A_1 \quad \overrightarrow{3R_1 + R_3 \to R_3} \quad B = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \\ -1 & -2 & 4 \end{bmatrix}$. Then $det(A)$ =

a) 8 b) 16 c) 4 d) cannot be determined

(v) In the previous question. Let E be an elementary matrix such that $EB = A_1$. Then $E =$

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(vi) in Question (iv). Let F be an elementary matrix such that $FA = A_1$. Then $F =$

a) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d) None of the previous is correct

(vii) In Question (iv), $det(2A_1)$ =

a) 8 b)16 c) 64 d) 4

(viii) Let A as in (iv). The solution to the system $AX = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$ is

a) $x_1 = -3, x_2 = 2, x_3 = 0$ b) $x_1 = -3.75, x_2 = \frac{4.75}{2}, x_3 = 0.25$

c) $x_1 = -2, x_2 = x_3 = 2$ d)None of the previous

(ix) Let A, B be invertible 10×10 matrices. Then one of the following statements is correct:

a) A is row-equivalent to B b) there are elementary 10×10 matrices, say $E_1, ..., E_n$, such that $E_1 E_2 \cdots E_n A = B$.

c) There is an invertible 10×10 matrix, say D, such that $DA = B$. d) All the previous are correct

(x) If A, B are 4×4 non-invertible matrices, then one of the following statement is correct.

a) A is row-equivalent to B. b) If $CA = B$, then $det(C) = 0$

c) A is not row-equivalent to B. d) None of the previous is correct.

(xi) Let A be a 4×4 matrix such that $det(A) = -3$. Let B be the third column of A. Then one of the following statement is correct:

a)It is possible to have $x_1 = x_2 = x_4 = -1$, and $x_3 = 4$ as a solution to the system $AX = B$.

b) It is possible that the system $AX = B$ has infinitely many solutions

c) $x_1 = x_2 = x_4 = 0$, $x_3 = 1$ is the only solution to the system $AX = B$.

d) More information is needed in order to solve $AX = B$.

(xii) Let A be a 2×2 matrix and $B = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$.Given $BA = C = \begin{bmatrix} a_1 + a_2 & 2a_2 \\ a_3 + a_4 & 2a_4 \end{bmatrix}$. Given $det(C) = 0$. Then $A^{-1} =$

a) Does not exist b) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. c) $\begin{bmatrix} 1 & 0 \\ -0.5 & 0.5 \end{bmatrix}$. d) None of the previous is correct

(xiii) let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$. Then A^{-1}

a) $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & -1 & -1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d) None of the above

(xiv) Let A as above. Then a solution to $AX = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ is

a) $x_1 = -1, x_2 = -1, x_3 = 0$ b)$x_1 = 2, x_2 = -2, x_3 = -1$

c) the system has infinitely many solution. d) none of the previous is correct

(xv) The values of k which make the system $\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & k \\ -2 & -2 & 2k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ consistent are

a) $k = -1$ b) $k \neq -1$

c) $k \neq -0.5$ and $k \neq -1$ d) There are no values for k that will make the system consistent

(xvi) The values of k which make the system $\begin{bmatrix} 4 & 0 & 2 \\ -2 & 0 & -1 \\ -4 & 0 & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ k \\ 4 \end{bmatrix}$ has infinitely many solution

a) $k = -2$ b) $k \neq -2$ c) $k \neq 2$ d)$k = 2$

(xvii) Assume the system in question (xvi) is consistent. Then a solution to the system is
a) $x_1 = -1, x_2 = 4.02, x_3 = 0$ b) $x_1 = -4, x_2 = x_3 = 0$

c) $x_1 = 0, x_2 = -1, x_3 = 0$ d) $x_1 = 4, x_2 = 0, x_3 = 2$.

(xviii) The solution to the system $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 0 & 4 \\ -2 & -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -6 \end{bmatrix}$ is

a. $x_1 = 0, x_2 = 2, x_3 = 0$ b. $x_1 = 1, x_2 = 1x_3 = 0$ c. $x_1 = 0, x_2 = 1, x_3 = -1$ d. The system has infinitely many solutions.

(xix) Given $A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 4 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ -1 & -1 & 2 \end{bmatrix}$. Let $C = (AB)^{-1}$. The (3, 2)-entry of C is

a)-2 b) 7 c) 4 d) None of the previous.

(xx) In the previous question, the (2, 3)-entry of B is

a) 0 b) 1 c) -1 d) 0.5 e) -0.5

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